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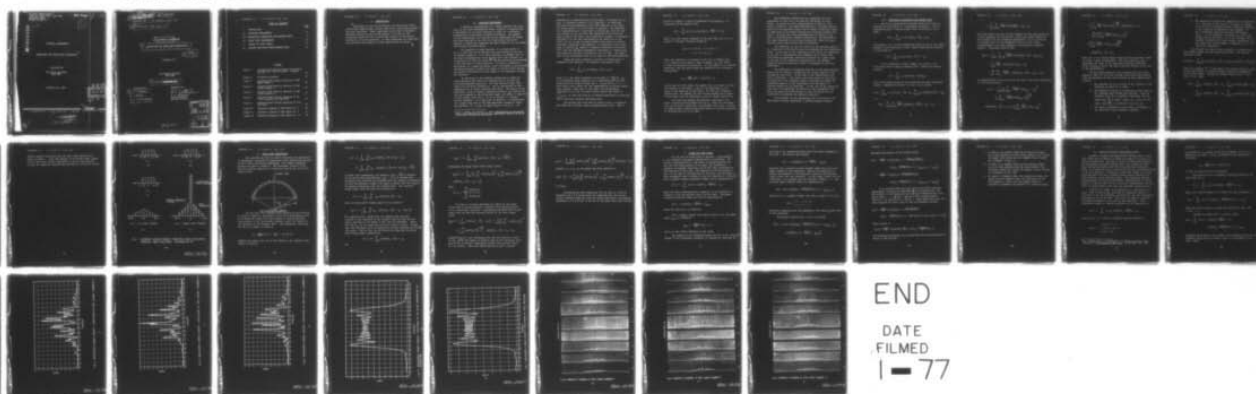
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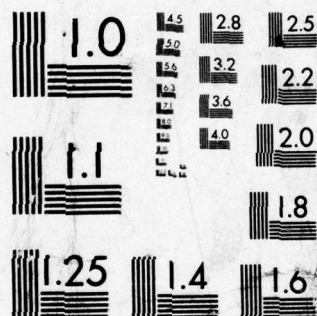
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TECHNICAL MEMORANDUM

ALGORITHMS FOR CODED PULSE PROCESSING ✓

Prepared for
The Bureau of Ships
Code 688E

February 14, 1964

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TRACOR

1701 GUADALUPE ST.

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10 Prepared by:
K. L./Oehler,
C./Tucker
A. F./Wittenborn

Approved by:
A. F. Wittenborn
Vice President
Project Director

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I. INTRODUCTION

↓ Algorithms for use in the analysis of coded pulse (band-width-pulse length product greater than unity) processing systems have been developed. These algorithms can be used to delineate desirable characteristics of the signals to be used in the processing system and are of particular utility in the digital simulation of system operation. In this memorandum the algorithms are described and examples of their applications are given. ↗

II. ALGORITHM DEVELOPMENT

A spectral representation of a finite duration time function, in its usual mathematical sense, consists of constructing a description of the given function in terms of a continuous distribution of sine waves of infinite duration. The members of this distribution are selected to have amplitudes and phases such that they add to zero for all times except during the interval in which they represent the given function, in accordance with Fourier transform theory.

For purposes of crude analysis, it is often useful to use as a unit the finite duration sine wave instead of the one of infinite duration. It is only necessary to remember that in the final interpretation of the results each line component of a spectrum is in fact the usual $\frac{\sin x}{x}$ type spectrum corresponding to the sine wave of finite duration. In order to formulate a description of a signal processor, such a procedure is particularly convenient because it leads very simply to results which display the spectral distribution of the signal as it progresses through the processor.

On the basis of the sampling theorem and by using the finite duration sine wave as a building block¹, we make the following plausible assertion: Any time function whose bandwidth is β and whose duration is τ can be represented by some combination of $\beta\tau$ finite duration sine waves. The choice of the amplitudes, relative phases, frequency interval, and duration of the waves depends on the particular signal one wishes to represent, as will become clear below. It is of interest to consider several examples: (a) a sine wave of duration τ is represented by a single sine wave of the same amplitude, frequency, phase, and duration; (b) a pulse of random noise of duration τ and

¹Alan A. Winder and Charles J. Loda, Introduction to Acoustical Space-time Information Processing, ONR Report ACR-63, pp 46-50.

bandwidth β is represented by $\beta\tau$ sine waves, all present at one time, at frequency intervals of $\frac{1}{\tau}$, duration τ , an amplitude distribution corresponding to the power spectrum of the pulse, and random phases; (c) an FM slide of duration τ and bandwidth β is represented by $\beta\tau$ sine waves of duration $\frac{1}{\beta}$, with one such wave present at any one time, the amplitude of each wave the same as that of the pulse, and occurring in sequence at frequency intervals $\frac{1}{\tau}$ in the order prescribed by the direction of the slide; and (d) a type of pseudo-random noise pulse of constant amplitude, bandwidth β and duration τ is represented by $\beta\tau$ sine waves of duration $\frac{1}{\beta}$, spaced at frequency intervals of multiples of $\frac{1}{\tau}$, with the different frequencies occurring in a random order such that only one wave is present at any one time. The amplitude of each incremental wave is set equal to the pulse amplitude.

Using the above concepts, a section of random noise of bandwidth β and duration τ is denoted by the general form

$$n(t) = \sum_{i=-M}^M n_i \cos [2\pi(f_0 + \frac{i}{\tau}) t + \phi_i]$$

where f_0 is the center frequency of the band, $M = \frac{\beta\tau - 1}{2}$, ϕ_i is an arbitrary phase angle, and it is understood that the sinusoidal waves all begin at some time $t = a$ and end at $t = a + \tau$. This form is most useful for the representation of CW and random noise signals. Note that in this form, the instantaneous amplitude of $n(t)$ may fluctuate widely between the limits of $\pm \sum_{i=-M}^M |n_i|$, so that this form is of limited usefulness in constructing a relatively constant amplitude pulse.

The second form to be used is better suited to represent FM slides and a certain class of pseudo-random noise. Here a

section of signal or noise of bandwidth β and duration τ is represented by the general form

$$a(t) = \sum_{i=1}^{\beta\tau} p(i,t) a_i \cos [2\pi(f_1 + \frac{m(i)}{\tau} t + \psi_i)]$$

where f_1 is the lowest frequency in the pass band, $p(i,t)$ is a "pulse" or "gating" function defined by

$$\begin{aligned} p(i,t) &= 0 \text{ if } \beta t < i-1 \text{ or } \beta t > i \\ &= 1 \text{ if } i-1 \leq \beta t < i \end{aligned}$$

where $m(i)$ denotes m , an integer in the set of numbers from 1 to $\beta\tau$ ordered on the index i , and the phases, ψ_i , are chosen so that the function is continuous. If the signal is of constant amplitude, then the phase angles are defined by the recurrence relationship

$$\psi_{i+1} = \frac{2\pi i}{\beta\tau} [m(i) - m(i+1)] + \psi_i.$$

In the case of an FM slide, the numbers $m(i)$ are chosen in order with respect to the index i , and all the amplitudes a_i are equal to the amplitude of the pulse. For the case of pseudo-random noise, the numbers $m(i)$ are chosen randomly with respect to the index i and the amplitudes a_i may be chosen arbitrarily.

Note that the maximum instantaneous amplitude of the pulse is given by the maximum value of a_i . Here the peak amplitude can be easily controlled, and therefore this form is useful in constructing a pseudo-random noise pulse of constant amplitude by choosing the a_i 's to be equal.

The differences between the two algorithms are very significant if they are to be used for the design of sonar pulses. By using the first representation in which $\beta\tau$ sine waves of duration τ are added simultaneously, the power spectral density of the generated signal can be closely controlled, while the amplitude of the signal envelope may fluctuate between wide limits. By using the second representation in which $\beta\tau$ sine waves of duration $\frac{1}{\beta}$ are combined sequentially, the amplitude of the signal envelope may be closely controlled, while the power spectral density may contain wide fluctuations.

The difference in ease of control of the envelope behavior as a function of time can be noted by examining the two formularizations. In the case of the simultaneous sine waves, the amplitude at any time is determined by the summation of $\beta\tau$ terms, each term having an amplitude coefficient modified by a phase angle. In the case of the sequential representation, the maximum amplitude during any period of time is determined by only one coefficient.

The difference in frequency behavior of the power spectral density can be noted by considering the voltage spectra of the incremental sine waves. In the case of the simultaneous sine waves of duration τ , the spectra are quite narrow so that when the spectrum of the composite signal is obtained by adding the individual spectra of each incremental wave the phases of adjacent spectra do not interfere. In the case of the sequential sine waves of duration $\frac{1}{\beta}$, the spectra of the incremental sine waves are quite wide and overlap, so that the phases must be considered in obtaining the composite spectrum.

The remainder of this memorandum will be devoted to the application of these algorithms in working example problems.

III. HETERODYNE CORRELATION WITH RANDOM NOISE

The first representation outlined above can be used to indicate the performance of a heterodyne correlator. Let the transmitted pulse of pseudo-random noise be denoted by $s(t)$, where

$$s(t) = \sum_{i=-M}^M s_i \cos \left[2\pi \left(f_o + \frac{i}{\tau} \right) t + \psi_i \right].$$

The replica, $r(t)$, of the transmitted pulse for use in the signal channel of the correlator is then, after inverting the spectrum of $s(t)$, about f_o ,

$$r(t) = \sum_{i=-M}^M r_i \cos \left[2\pi \left(f_o - \frac{i}{\tau} \right) t - \psi_i \right].$$

It is convenient to first examine the action of the correlator with an input of random noise. Denote the received noise by

$$n(t) = \sum_{i=-M}^M n_i \cos \left[2\pi \left(f_o + \frac{i}{\tau} \right) t + \varphi_i \right].$$

where the received noise is not correlated with the transmitted signal. Forming the product $r(t) n(t) = G(t)$ yields

$$G(t) = \sum_{i=-M}^M r_i \cos \left[2\pi \left(f_o - \frac{i}{\tau} \right) t - \psi_i \right] \sum_{k=-M}^M n_k \cos \left[2\pi \left(f_o + \frac{k}{\tau} \right) t + \varphi_k \right].$$

or,

$$G(t) = \sum_{i=-M}^M \sum_{k=-M}^M \frac{r_i n_k}{2} \cos \left[2\pi \left(2f_o + \frac{k-i}{\tau} \right) t + \varphi_k - \psi_i \right] +$$

$$+ \sum_{i=-M}^M \sum_{k=-M}^M \frac{r_i n_k}{2} \cos[2\pi(\frac{k+i}{\tau})t + \varphi_k + \psi_i].$$

The first double sum term of $G(t)$ contains all the sum frequencies of the components of $r(t)$ and $n(t)$, while the second double sum term contains all the difference frequencies. It is convenient to consider only the sum frequencies and to collect terms corresponding to the same frequency by altering the summation indices. Collecting terms we have

$$\begin{aligned} G_s(t) = & \sum_{j=-2M}^{-1} \sum_{i=-j-M}^M \frac{r_i n_{i+j}}{2} \cos[2\pi(2f_o + \frac{j}{\tau})t + \varphi_{i+j} - \psi_i] \\ & + \sum_{i=-M}^M \frac{r_i n_i}{2} \cos[2\pi(2f_o) t + \varphi_i - \psi_i] \\ & + \sum_{j=1}^{2M} \sum_{i=-M}^{M-j} \frac{r_i n_{i+j}}{2} \cos[2\pi(2f_o + \frac{j}{\tau})t + \varphi_{i+j} - \psi_i]. \end{aligned}$$

By expanding the cosine in each of the sums in the above expression, it can be rewritten as

$$\begin{aligned} G_s(t) = & \sum_{j=-2M}^{-1} \left\{ \left[\sum_{i=-M-j}^M \frac{r_i n_{i+j}}{2} \cos(\varphi_{i+j} - \psi_i) \right]^2 \right. \\ & \left. + \left[\sum_{i=-M-j}^M \frac{r_i n_{i+j}}{2} \sin(\varphi_{i+j} - \psi_i) \right]^2 \right\}^{\frac{1}{2}} \\ & \cos[2\pi(2f_o + \frac{j}{\tau}) t + \varphi_j] + \left\{ \left[\sum_{i=-M}^M \frac{r_i n_i}{2} \cos(\varphi_i - \psi_i) \right]^2 \right. \\ & \left. + \left[\sum_{i=-M}^M \frac{r_i n_i}{2} \sin(\varphi_i - \psi_i) \right]^2 \right\}^{\frac{1}{2}} \cos[2\pi(2f_o) t + \varphi_0] \end{aligned}$$

$$\begin{aligned}
& + \left[\sum_{i=-M}^M \frac{r_i n_i}{2} \sin(\varphi_i - \psi_i) \right]^2 \Bigg\}^{\frac{1}{2}} \cos[2\pi(2f_0)t + \alpha_0] \\
& + \sum_{j=1}^{2M} \left\{ \left[\sum_{i=-M}^{M-j} \frac{r_i n_{i+j}}{2} \cos(\varphi_{i+j} - \psi_i) \right]^2 \right. \\
& \left. + \left[\sum_{i=-M}^{M-j} \frac{r_i n_{i+j}}{2} \sin(\varphi_{i+j} - \psi_i) \right]^2 \right\}^{\frac{1}{2}} \\
& \cos[2\pi(2f_0 + \frac{j}{\tau}) + \theta_j]
\end{aligned}$$

where the θ_j are constant phase angles for the particular sample of length τ under consideration. The above function, $G_s(t)$ represents the input to the heterodyne correlator band-pass filter located at $2f_0$ (or the bank of doppler filters in the case of the SQS-26(XN-2)).

The following properties can be deduced from the above equation, where the list also includes those which were obvious from the beginning and have been verified by more sophisticated analysis.

1. The component of $G_s(t)$ located at $2f_0$ consists of the summation of $(2M+1)$ or $\beta\tau$ terms.
2. The summations consist of successively fewer terms as the frequency gets further away from $2f_0$. Thus for frequency $(2f_0 \pm \frac{1}{\tau})$ it consists of $2M$ terms, for frequency $(2f_0 \pm \frac{2}{\tau})$ of $(2M-1)$ terms, etc., until for frequency $(2f_0 \pm \frac{2M}{\tau})$ it consists of only one term.
3. All useful information at this point is contained in the amplitude of each frequency component.

4. The amplitude of each of the components in the spectrum, i.e., the value of the sum on i for given j , depends on the values of the epoch angles, completely analogous to the situation encountered in beam forming. If $2M$ is reasonably large and the phase differences $(\phi_{i+j} - \psi_i)$ are random as a function of i , the value of each sum on the index i is, on the average, for all possible combinations of epoch angles, proportional to the square root of the number of components, i.e., the terms sum incoherently.

Now consider the input of the correlator to contain both a zero doppler signal and the uncorrelated noise. The received waveform is then

$$n(t)+s(t) = \sum_{i=-M}^M \left\{ n_i \cos[2\pi(f_0 + \frac{i}{\tau})t + \phi_i] + s_i \cos[2\pi(f_0 + \frac{i}{\tau})t + \psi_{is}] \right\}$$

where the subscript "s" on the phase of the signal portion is used to note the effects of possible changes between the transmitted and received signal. Forming the product function $G(t) = r(t)[n(t) + s(t)]$ gives

$$G(t) = \sum_{i=-M}^M r_i \cos[2\pi(f_0 - \frac{i}{\tau})t - \psi_i] \sum_{k=-M}^M n_k \cos[2\pi(f_0 + \frac{k}{\tau})t + \phi_k] \\ + \sum_{i=-M}^M r_i \cos[2\pi(f_0 - \frac{i}{\tau})t - \psi_i] \sum_{k=-M}^M s_k \cos[2\pi(f_0 + \frac{k}{\tau})t + \psi_{ks}]$$

or

$$\begin{aligned}
G(t) = & \sum_{i=-M}^M \sum_{k=-M}^M \left\{ \frac{r_{i n k}}{2} \cos[2\pi(f_0 + \frac{k-i}{\tau})t + \varphi_k - \psi_i] \right. \\
& + \left. \frac{r_{i s k}}{2} \cos[2\pi(2f_0 + \frac{k-i}{\tau})t + \psi_{ks} - \psi_i] \right\} \\
& + \sum_{i=-M}^M \sum_{k=-M}^M \left\{ \frac{r_{i n k}}{2} \cos[2\pi(\frac{i+k}{\tau})t + \varphi_k + \psi_i] \right. \\
& + \left. \frac{r_{i s k}}{2} \cos[2\pi(\frac{i+k}{\tau})t + \psi_{ks} - \psi_i] \right\}
\end{aligned}$$

where the first double sum contains all the sum frequencies and the second double sum contains all the difference frequencies. It is again convenient to consider only the sum frequencies and to collect terms corresponding to the same frequency.

Collecting terms, we have for the sum frequency

$$\begin{aligned}
G_s(t) = & \sum_{j=-2M}^{-1} \sum_{i=-j-M}^M \left\{ \frac{r_{i n i+j}}{2} \cos[2\pi(2f_0 + \frac{j}{\tau})t + \varphi_{i+j} - \psi_i] \right. \\
& + \left. \frac{r_{i s i+j}}{2} \cos[2\pi(2f_0 + \frac{j}{\tau})t + \psi_{(i+j)s} - \psi_i] \right\} \\
& + \sum_{i=-M}^M \left\{ \frac{r_{i n i}}{2} \cos[2\pi(2f_0)t + \varphi_i - \psi_i] + \frac{r_{i s i}}{2} \cos[2\pi(2f_0)t + \psi_{is} - \psi_i] \right\} \\
& + \sum_{j=1}^{2M} \sum_{i=-M}^{M-j} \left\{ \frac{r_{i n i+j}}{2} \cos[2\pi(2f_0 + \frac{j}{\tau})t + \varphi_{i+j} - \psi_i] \right. \\
& + \left. \frac{r_{i s i+j}}{2} \cos[2\pi(2f_0 + \frac{j}{\tau})t + \psi_{(i+j)s} - \psi_i] \right\} .
\end{aligned}$$

The component located at $2f_0$ is of particular interest because it is the frequency which excites the heterodyne correlator band-pass filter. Denoting this term by $G_f(t)$, we have

$$G_f(t) = \sum_{i=-M}^M \left\{ \frac{r_i n_i}{2} \cos[4\pi f_0 t + \varphi_i - \psi_i] + \frac{r_i s_i}{2} \cos[4\pi f_0 t + \psi_{is} - \psi_i] \right\}.$$

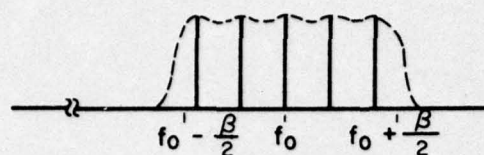
Rearranging we have

$$G_f(t) = \left\{ \left[\sum_{i=-M}^M \frac{r_i n_i}{2} \cos(\varphi_i - \psi_i) \right]^2 + \left[\sum_{i=-M}^M \frac{r_i n_i}{2} \sin(\varphi_i - \psi_i) \right]^2 \right\}^{\frac{1}{2}} \\ \cos[4\pi f_0 t + \theta_0] \\ + \left\{ \left[\sum_{i=-M}^M \frac{r_i s_i}{2} \cos(\psi_{is} - \psi_i) \right]^2 + \left[\sum_{i=-M}^M \frac{r_i s_i}{2} \sin(\psi_{is} - \psi_i) \right]^2 \right\}^{\frac{1}{2}} \cos[4\pi f_0 t + \alpha_0]$$

where θ_0 and α_0 are constant phase angles for the particular sample of noise of length τ under consideration.

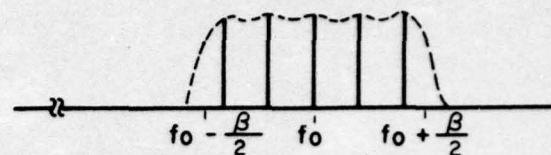
Examining the above equation yields an indication of the processing gain of the heterodyne correlator. The output component at $2f_0$ due to the noise sums incoherently because of the random nature of the phase difference $(\varphi_i - \psi_i)$ and thus the output due to noise is approximately proportional to the square root of the number of terms (or $\sqrt{\beta\tau}$ since there are $\beta\tau$ terms). The output component due the signal sums coherently because the phase difference $(\psi_{is} - \psi_i)$ is approximately zero. Hence the signal output is proportional to the number of terms ($\beta\tau$). Thus the amplitude of the component at frequency $2f_0$ due to a signal is greater, by a factor proportional to the square root of the pulse duration-bandwidth product, than when noise alone is present at the same power level. The power

density spectra associated with the above time functions are shown in Figure 1. Note that because of the finite pulse length, each of the lines actually represents a $\sin x/x$ type spectrum so that the composite spectrum over the band is relatively smooth.



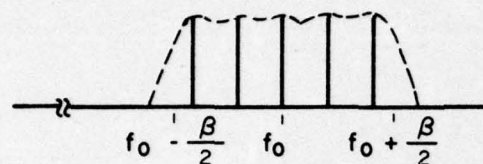
$s(t)$

(A)



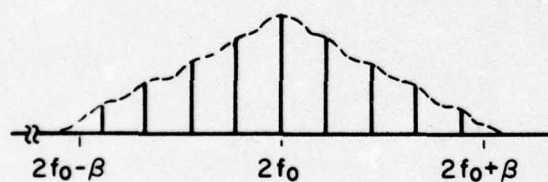
$n(t)$

(B)



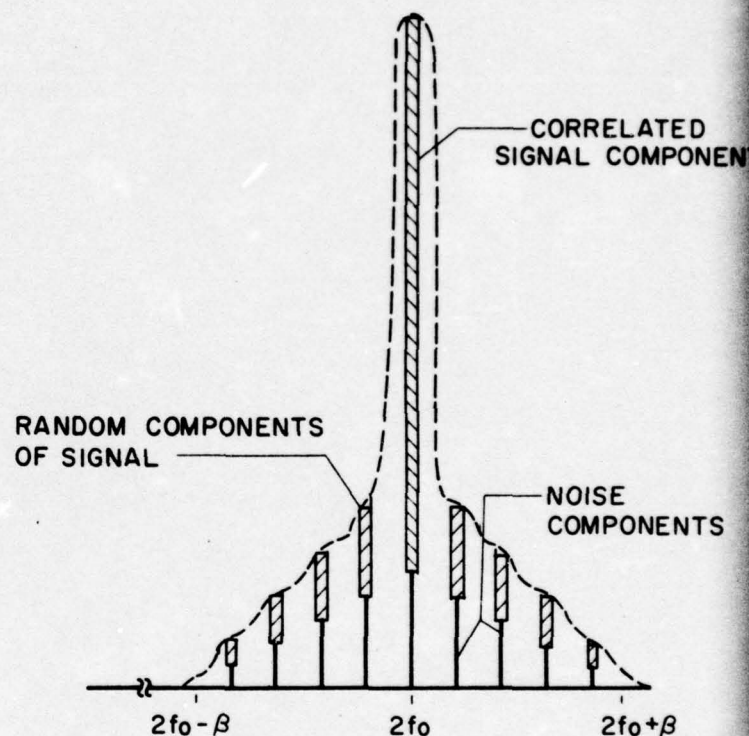
$r(t)$

(C)



$G_s(t)$ (NO SIGNAL PRESENT)

(D)



$G_s(t)$ (SIGNAL = NOISE PRESENT)

(E)

Fig. 1 - AVERAGE POWER DENSITY SPECTRA ASSOCIATED WITH VARIOUS TIME FUNCTIONS. ASSUMED $\beta\tau = 5$

12a

IV. PHASE SHIFT BEAMFORMING

The algorithm may also be used to determine the degradation in performance introduced by using phase-shift beamforming instead of the correct time-delay beamforming. The example transducer geometry is shown in Figure 2. The transducer consists of 24 vertical elements spaced at 5° intervals along an arc of an 8 ft. radius circle.

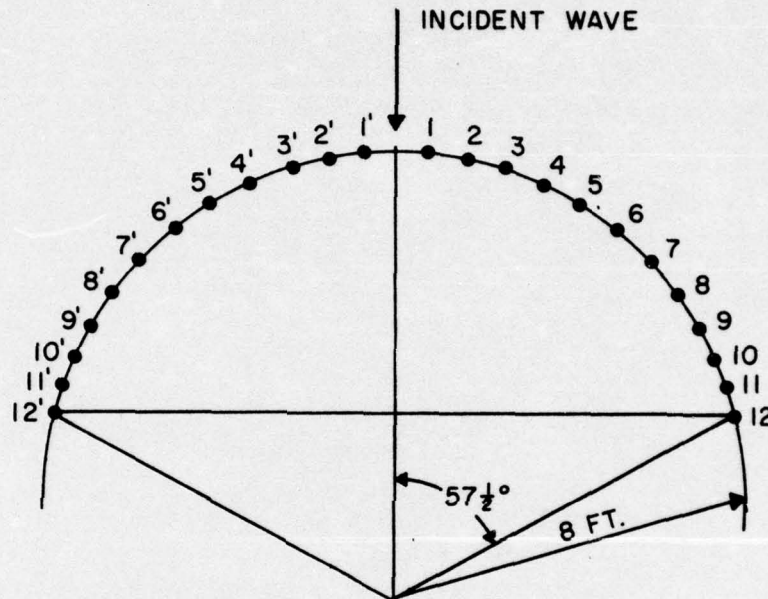


Fig. 2 - TRANSDUCER GEOMETRY

For the case of an incident wave arriving in the direction as shown in Figure 2, one method of time delay beamforming would be to delay the output from each element to obtain equivalent time arrival at the line joining elements 12 and 12' (normal to the direction of the incident wave). The appropriate time delay for the e 'th element is given by

$$t_e = \frac{8}{5000} \left[\cos \left(e - \frac{1}{2} \right) 5^\circ - \cos 57.5^\circ \right].$$

Summing the signals over all of the elements, the composite beam signal is given by

$$s(t) = 2 \sum_{i=-M}^M \sum_{e=1}^{12} s_{ie} \cos[2\pi(f_o + \frac{i}{\tau})(t-t_e) + \psi_i]$$

$$= 2 \sum_{i=-M}^M \sum_{e=1}^{12} s_{ie} \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_i + 2\pi[t_e f_o + \frac{t_e i}{\tau}]] .$$

In phase shift beamforming, the quantity $(t_e f_o + \frac{t_e i}{\tau})$ is replaced by its narrow band approximation, $t_e f_o$. If it is assumed that the phase shift networks are designed such that signals of frequency f_o from the different elements are added in phase, then the signals of adjacent frequencies will be in phase error. The correctly formed signal from the array will be given by

$$s(t) = 2 \sum_{i=-M}^M \sum_{e=1}^{12} s_{ie} \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_i]$$

while the phase-shift formed signal will be given by

$$s_{\phi}(t) = 2 \sum_{i=-M}^M \sum_{e=1}^{12} s_{ie} \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_n + 2\pi t_e \frac{i}{\tau}].$$

For simplification, assume that all elements have the same response in the direction of the incident beam and that the frequency spectrum of the signal is flat over the bandwidth. Therefore the s_{ie} are all equal and can be set equal to unity. Further note that the phase error is doubled for two way transmission through the system. Then we have for the correctly formed and the phase-shift formed signal respectively

$$s(t) = 24 \sum_{i=-M}^M \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_i]$$

and

$$s_{2\phi}(t) = 2 \sum_{i=-M}^M \sum_{e=1}^{12} \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_i + \frac{4\pi t_e i}{\tau}] .$$

Rearranging the phase-shift formed signal yields

$$s_{2\phi}(t) = 2 \sum_{i=-M}^M \left\{ \left[\sum_{e=1}^{12} \cos(4\pi t_e \frac{i}{\tau}) \right]^2 + \left[\sum_{e=1}^{12} \sin(4\pi t_e \frac{i}{\tau}) \right]^2 \right\}^{\frac{1}{2}} \cos \left[2\pi(f_o + \frac{i}{\tau})t + \psi_i + \theta_i \right]$$

where

$$\tan \theta_i = \frac{\sum_{e=1}^{12} \sin(4\pi t_e \frac{i}{\tau})}{\sum_{e=1}^{12} \cos(4\pi t_e \frac{i}{\tau})} .$$

In order to actually determine the effect of the phase-formed beam on system operation, consider the processing system to be a heterodyne correlator. Multiplying the phase-shift formed signal with the spectrum inverted replica of the exact signal yields

$$G_{2\phi}(t) = \left\{ \sum_{i=-M}^M \cos[2\pi(f_o - \frac{i}{\tau})t - \psi_i] \right\}^2 \sum_{i=-M}^M \left\{ \left[\sum_{e=1}^{12} \cos(4\pi t_e \frac{i}{\tau}) \right]^2 + \left[\sum_{e=1}^{12} \sin(4\pi t_e \frac{i}{\tau}) \right]^2 \right\}^{\frac{1}{2}} \cos[2\pi(f_o + \frac{i}{\tau})t + \psi_i + \theta_i] .$$

Expanding $G_{2\phi}(t)$, and considering only the sum frequency located at $2f_o$, yields the signal as would be seen by the correlator band-pass filter at perfect registration between the received signal and the replica. Then this signal denoted by $G_f(t)$, becomes

$$G_f(t) = \sum_{i=-M}^M \left\{ \left[\sum_{e=1}^{12} \cos(4\pi t_e \frac{i}{\tau}) \right]^2 + \left[\sum_{e=1}^{12} \sin(4\pi t_e \frac{i}{\tau}) \right]^2 \right\}^{\frac{1}{2}} \cos(4\pi f_0 t + \theta_n).$$

Because $\theta_i = -\theta_{-i}$, we can reduce the above equation to

$$G_f(t) = \left[1 + 2 \sum_{i=1}^M \left\{ \left[\sum_{e=1}^{12} \cos(4\pi t_e \frac{i}{\tau}) \right]^2 + \left[\sum_{e=1}^{12} \sin(4\pi t_e \frac{i}{\tau}) \right]^2 \right\}^{\frac{1}{2}} \cos \theta_i \right]$$

$$\cos 4\pi f_0 t.$$

Evaluating the above expression for the case of a 100 cps bandwidth and a 0.5 second pulse and comparing with the similar results obtained using the correctly formed beam shows a performance degradation of 0.2 db due to the use of the phase-shift beam forming.

V. LINEAR FM SLIDE SIGNAL

For the case of an FM slide of duration τ and bandwidth β , the sequential frequency representation is preferred. The slide is represented by $\beta\tau$ sine waves of duration $\frac{1}{\beta}$ with only one such wave being present at one time, and occurring in sequence at frequency intervals of $\frac{1}{\tau}$ in the order prescribed by the direction of the slide. The amplitude of each incremental wave is the same as the amplitude of the pulse, and the phase of each incremental wave is chosen so that the function is continuous. Thus an upward sliding transmitted signal can be represented by

$$s(t) = a \sum_{i=1}^{\beta\tau} \rho(i,t) \cos[2\pi(f_1 + \frac{m(i)}{\tau})t + \psi_i].$$

While the above representation is general, it is convenient to simplify it for the specific case of an FM slide. The upward sliding transmitted signal can also be represented by

$$s(t) = a \cos[2\pi(f_1 + \frac{k(t)}{\tau})t + \psi_{k(t)}]$$

where the index $k(t)$ is defined by

$k(t) \equiv$ largest integer less than or equal to βt , the phase angle $\psi_{k(t)}$ is defined by

$$\psi_{k(t)} = - \frac{\pi k(t)}{\beta\tau} [k(t) + 1],$$

and f_0 is the initial frequency of the slide.

The replica of the transmitted pulse for use in the reference channel of the heterodyne correlator is obtained by inverting the

spectrum of the transmitted pulse about the center frequency of the slide. The replica then becomes

$$r(t) = a \cos[2\pi(f_1 + \beta - \frac{k(t)}{\tau})t - \psi_k(t)].$$

Generally only a portion of the returned signal will be in the signal channel of the correlator. Thus it is instructive to examine the behavior of the correlator output as the received signal slides into the correlator. As the signal slides into the correlator, it is present only for duration σ where $\sigma < \tau$. The received signal (without noise) is represented by

$$s(t) = b(t) \cos[2\pi(f_1 + \frac{k(t+\sigma-\tau)}{\tau})(t+\sigma-\tau) + \psi_k(t+\sigma-\tau)]$$

where $K(t+\sigma-\tau) \equiv$ largest integer less than or equal to $(t+\sigma-\tau)\beta$,

$$b(t) = \begin{cases} b, & \tau - \sigma \leq t \leq \tau \\ 0, & t < \tau - \sigma, \end{cases}$$

and time is measured from the beginning of the replica pulse used in the correlator.

The product function $G(t) = r(t) s(t)$ yields

$$G(t) = a b(t) \cos \left\{ 2\pi \left[f_1 + \frac{k(t+\sigma-\tau)}{\tau} \right] (t+\sigma-\tau) + \psi_k(t+\sigma-\tau) \right\}$$

$$\cos \left\{ 2\pi \left[f_1 + \beta - \frac{k(t)}{\tau} \right] t - \psi_k(t) \right\}.$$

Expanding the product of the cosines yields

$$\begin{aligned}
 G(t) = & \frac{ab(t)}{2} \cos \left\{ 2\pi \left[2f_1 + \beta + \frac{K(t+\sigma-\tau)-k(t)}{\tau} \right] t \right. \\
 & \left. + 2\pi \left[f_1 + \frac{k(t+\sigma-\tau)}{\tau} \right] (\sigma-\tau) + \psi_{k(t+\sigma-\tau)} - \psi_{k(t)} \right\} \\
 & + \frac{ab(t)}{2} \cos \left\{ 2\pi \left[-\beta + \frac{k(t)+k(t+\sigma-\tau)}{\tau} \right] t \right. \\
 & \left. + 2\pi \left[f_1 + \frac{k(t+\sigma-\tau)}{\tau} \right] (\sigma-\tau) + \psi_{k(t+\sigma-\tau)} + \psi_{k(t)} \right\} .
 \end{aligned}$$

It is asserted that the first term of the above equation approximates a sinusoid of amplitude $\frac{ab}{2}$ and frequency $2f_1 + \frac{\sigma\beta}{\tau}$. The validity of this assertion can readily be demonstrated at the points where $\tau - \sigma = \frac{m}{\beta}$ and $t = \frac{n}{\beta}$ with m and n being integers. Here $k(t) = \beta t$ and $k(t+\sigma-\tau) = \beta(t+\sigma-\tau)$. When substituted into the above equation the first term (or the sum frequency) becomes

$$\begin{aligned}
 G_s(t) = & \frac{ab(t)}{2} \cos \left\{ 2\pi \left[2f_1 + \beta \frac{(t+\sigma-\tau)\beta - \beta t}{\tau} \right] t \right. \\
 & \left. + 2\pi \left[f_1 + \frac{(t+\sigma-\tau)}{\tau} \beta \right] (\sigma-\tau) + \frac{\pi t}{\tau} (\beta t + 1) - \frac{\pi}{\tau} (t+\sigma-\tau) [\beta(t+\sigma-\tau) + 1] \right\}
 \end{aligned}$$

which reduces to

$$G_s(t) = \frac{ab(t)}{2} \cos \left[2\pi \left(2f_1 + \frac{\beta\sigma}{\tau} \right) t + 2\pi \left(f_1 + \frac{\beta\sigma - \beta\tau - 1}{2\tau} \right) (\sigma - \tau) \right].$$

The following conclusions can be drawn from the preceding representation of the FM slide.

1. If there is no doppler shift and the received signal is in time registry with the stored replica, the correlator output will excite the zero doppler filter located at a frequency of $2f_1 + \beta$.
2. If there is no doppler shift and the received signal is not in time registry with the stored replica, the correlator output will excite the doppler filter located at a frequency of $2f_1 + \frac{\sigma}{\tau}\beta$.
3. If there is a doppler shift, the correlator output will excite the zero doppler filter for some value of the time registry σ ; hence only one doppler filter is required to determine the presence of a received signal.

VI. UNIFORM AMPLITUDE PSEUDO-RANDOM NOISE

Studies by Stewart and Westerfield² indicate that a noise-like signal may be superior to either FM slides or CW signals in several applications. The use of an actual random noise signal is not particularly desirable because most sonar systems are peak-power limited and random noise has large amplitude variations. While it is relatively easy to obtain noise of the desired bandwidth, it has been difficult to generate a noise-like signal of constant amplitude which also has a uniform energy density spectrum over each relatively short duration signal. The sequential frequency representation (of which the preceding representation of the FM slide is a special case) is well suited for representing a certain class of pseudo-random noise which has desirable characteristics.

Consider the signal to be composed of $\beta\tau$ incremental sine waves, each of duration $\frac{1}{\beta}$, occurring at frequencies spaced at intervals $\frac{1}{\tau}$ over the bandwidth, and with only one such incremental wave present at any one time. Note that the order in which the individual frequencies occur has not been specified. Thus a transmitted signal of constant amplitude can be represented by

$$S(t) = a \sum_{i=1}^{\beta\tau} \rho(i,t) \cos[2\pi(f_1 + \frac{m(i)}{\tau})t + \psi_i]$$

where $p(i,t)$ is a "pulse" or "gating" function defined by

$$\begin{aligned} & 0 \text{ if } \beta t < i - 1 \\ p(i,t) &= 1 \text{ if } i-1 \leq \beta t < i \\ & 0 \text{ if } i < \beta t \end{aligned}$$

²J.L. Stewart and E.C. Westerfield, "A Theory of Active Sonar Detection," Proceedings of the I.R.E., pp 872-881; May, 1959.

$m(i)$ denotes m , an integer in the set of numbers from 1 to $\beta\tau$ ordered on the index i , and ψ_i is defined by the recurrence relationship

$$\psi_{i+1} = \frac{2\pi i}{\beta\tau} [m(i) - m(i+1)] + \psi_i$$

so that the function $s(t)$ is continuous.

The replica of $s(t)$ is formed by inverting frequency and phase about f_1 , so

$$r(t) = \sum_{i=1}^{\beta\tau} \rho(i,t) \cos[2\pi(f_1 - \frac{m(i)}{\tau})t - \psi_i].$$

Denote the received signal as it slides into the correlator by

$$s(t) = \sum_{i=j}^{\beta\tau} \rho(i,t) \cos[2\pi(f_1 + \frac{m(i-j)}{\tau})(t+\sigma-\tau) + \psi_{i-j}]$$

where the received signal overlaps the replica for time σ and $j = \beta(\tau - \sigma)$.

Forming the product $G(t) = s(t)r(t)$ yields

$$G(t) = \sum_{i=j}^{\beta\tau} \rho(i,t) \cos[2\pi(f_1 - \frac{m(i)}{\tau})t - \psi_i]$$

$$\cos[2\pi(f_1 + \frac{m(i-j)}{\tau})(t+\sigma-\tau) + \psi_{i-j}].$$

Expanding the product of the cosines, and considering only the sum frequency terms yields the input, $G_s(t)$, to the doppler filter bank. Hence

$$G_s(t) = \sum_{i=j}^{\beta\tau} \rho(i,t) \cos[2\pi(2f_0 + \frac{m(i-j)-m(i)}{\tau})t + \psi_{i-j} - \psi_i + 2\pi(f_1 + \frac{m(i-j)}{\tau})(\sigma-\tau)].$$

Considering the range-range rate ambiguity and examining the above equation indicates the most desirable type of signal which can be generated using this representation. Consider the following types of signal generated by the above procedure.

(1) Assume that all of the m 's are equal. This reduces to the case of a CW signal which has good range resolution but poor frequency resolution if τ is short, and poor range resolution but good frequency resolution if τ is long.

(2) Assume that the m 's are chosen in a linear sequence. This reduces to the case of an FM slide which possesses a range-range rate ambiguity in which range can be determined accurately only if the range rate is known and range rate can be determined accurately only if the range is known.

(3) Assume that the m 's are chosen randomly. This causes the frequency shift $\frac{m(i-j)-m(i)}{\tau}$ to be a random function spread over a range of 2β unless $j = 0$ when there will be no frequency shift. Hence the signal is capable of good resolution in both range and range rate. As previously noted, the signal is very desirable because it retains the constant amplitude property.

The algorithm described in this section has been used to generate several constant amplitude PRN signals. The characteristics of these signals were investigated by measuring their power density spectra and by measuring various correlation functions.

The measured power density spectra of three PRN signals are shown in Figures 3-5. In each case, the signal had constant amplitude over its duration of $1/2$ second and was intended to have a flat frequency spectrum over the range of 100 cps to 200 cps. While the signals did not have the desired rectangular power spectra, they did have the essential characteristics for use as sonar signals. Very little signal power was outside of the desired bandwidth and the power was distributed over the entire bandwidth, even though the distribution was not as uniform as could be desired. For comparison, the power density spectra of two linear FM slides are shown. The spectrum shown in Figure 6 was obtained for a slide which was generated by using the algorithm of this section, while the slide used to obtain Figure 7 was generated by using the true mathematical representation of a linear slide.

The correlation functions of these pseudo-random noise signals are shown in Figures 8-10. The set of correlation functions shown in each figure actually represent sections sliced from an ambiguity diagram with all of the slices being parallel to the time axis. The nine different bands of each figure represent the polarity cross-correlation functions of the original signal against a replica of the signal which has been modified by shifting it by a constant frequency increment. The nine frequency increments used are 8, 6, 4, 2, 0, -2, -4, -6, and -8 cps. The center band represents zero frequency shift and thus is the auto-correlation function.

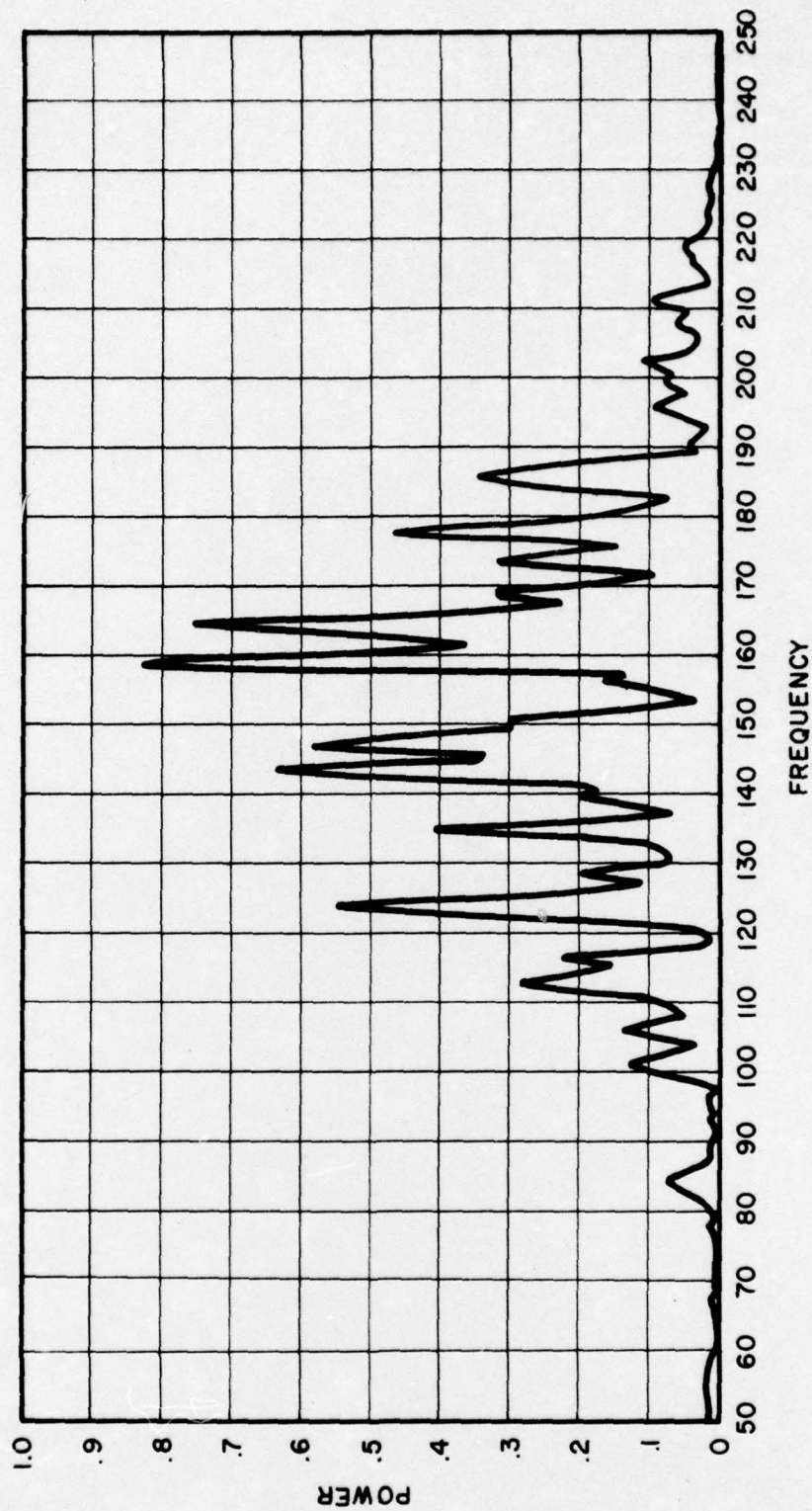


Fig. 3 - MEASURED POWER DENSITY SPECTRUM OF PRN SIGNAL NUMBER 4

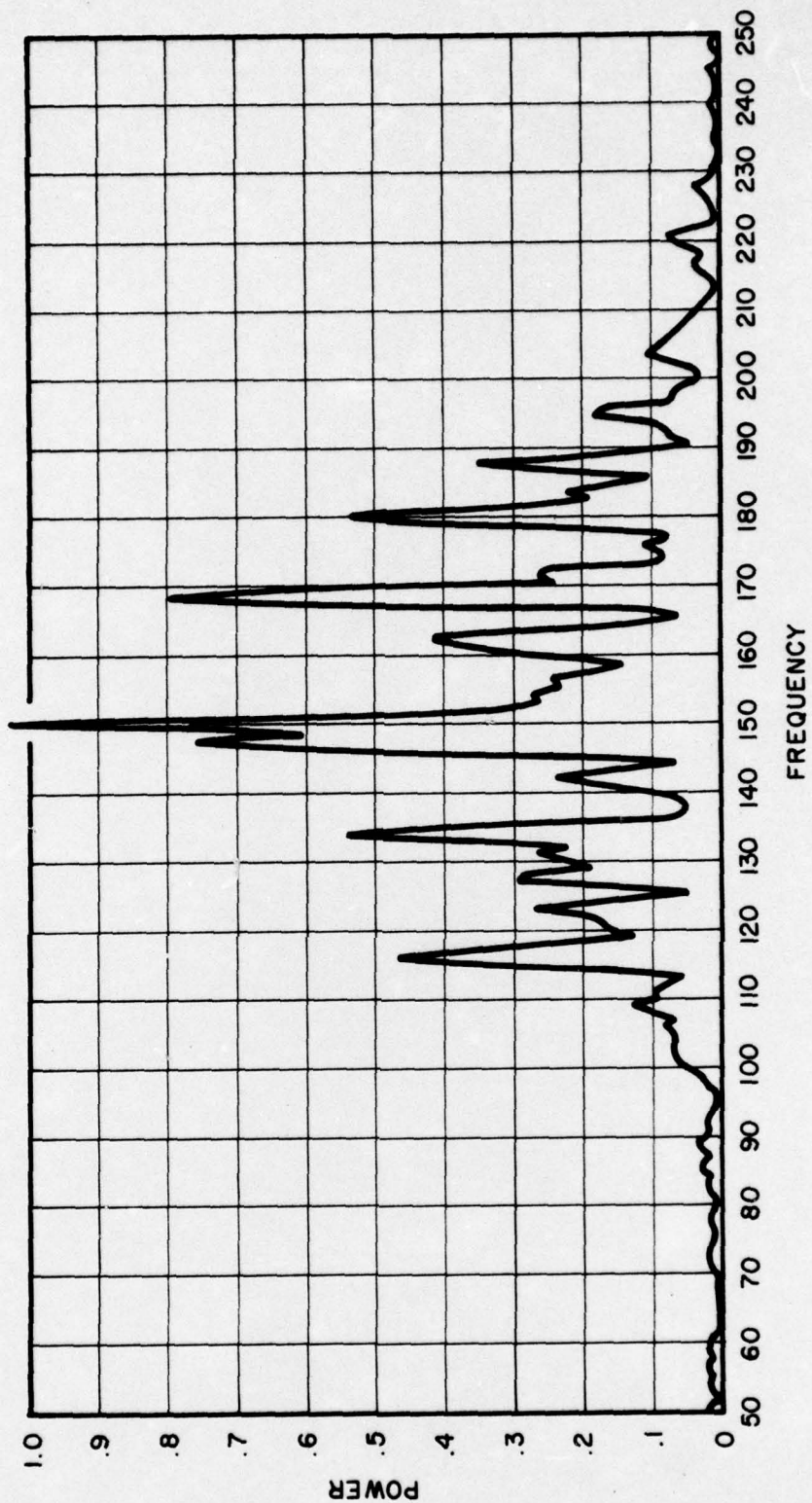


Fig. 4 - MEASURED POWER DENSITY SPECTRUM OF PRN SIGNAL NUMBER 6

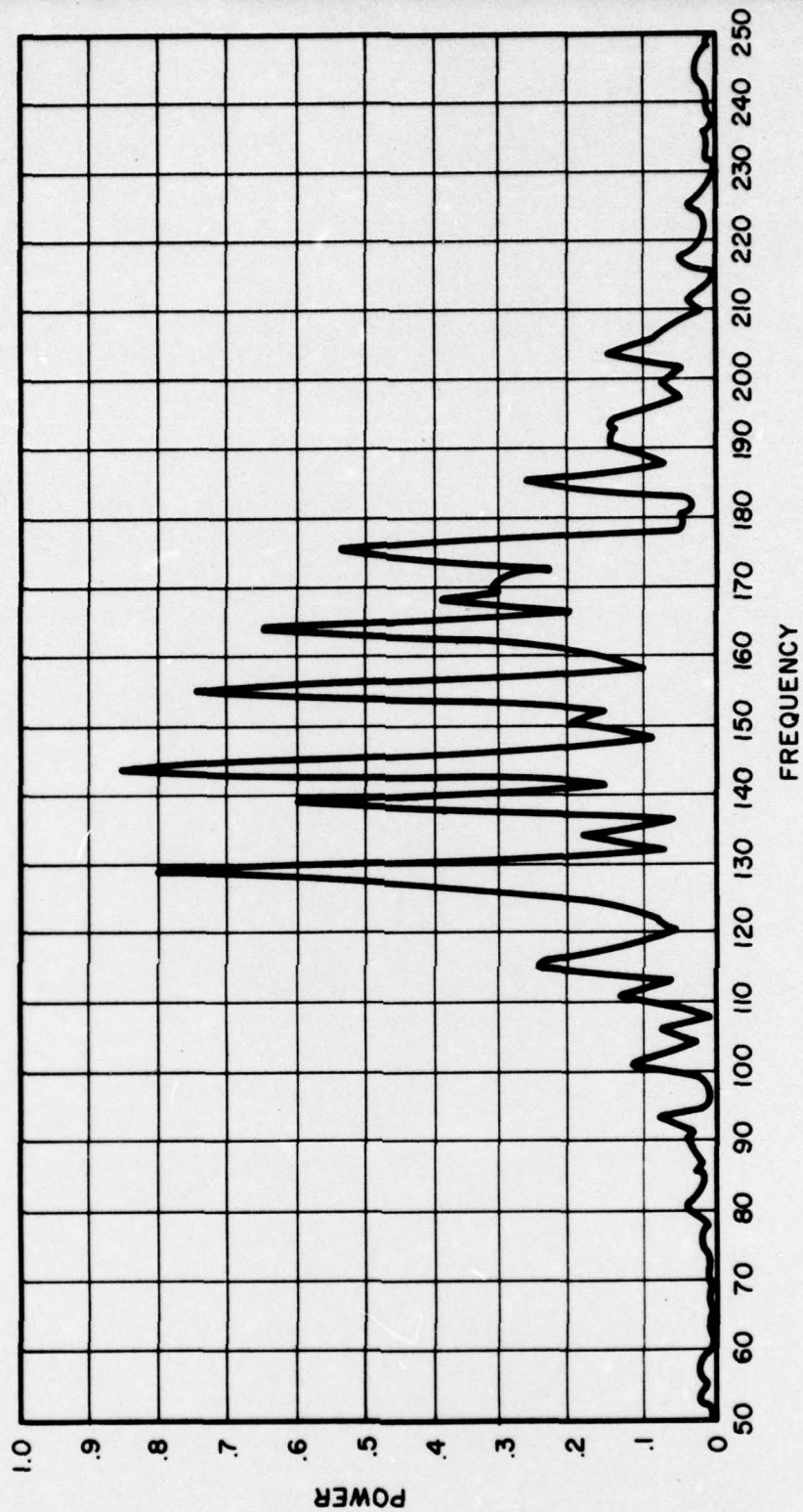


Fig. 5 - MEASURED POWER DENSITY SPECTRUM OF PRN SIGNAL NUMBER 12

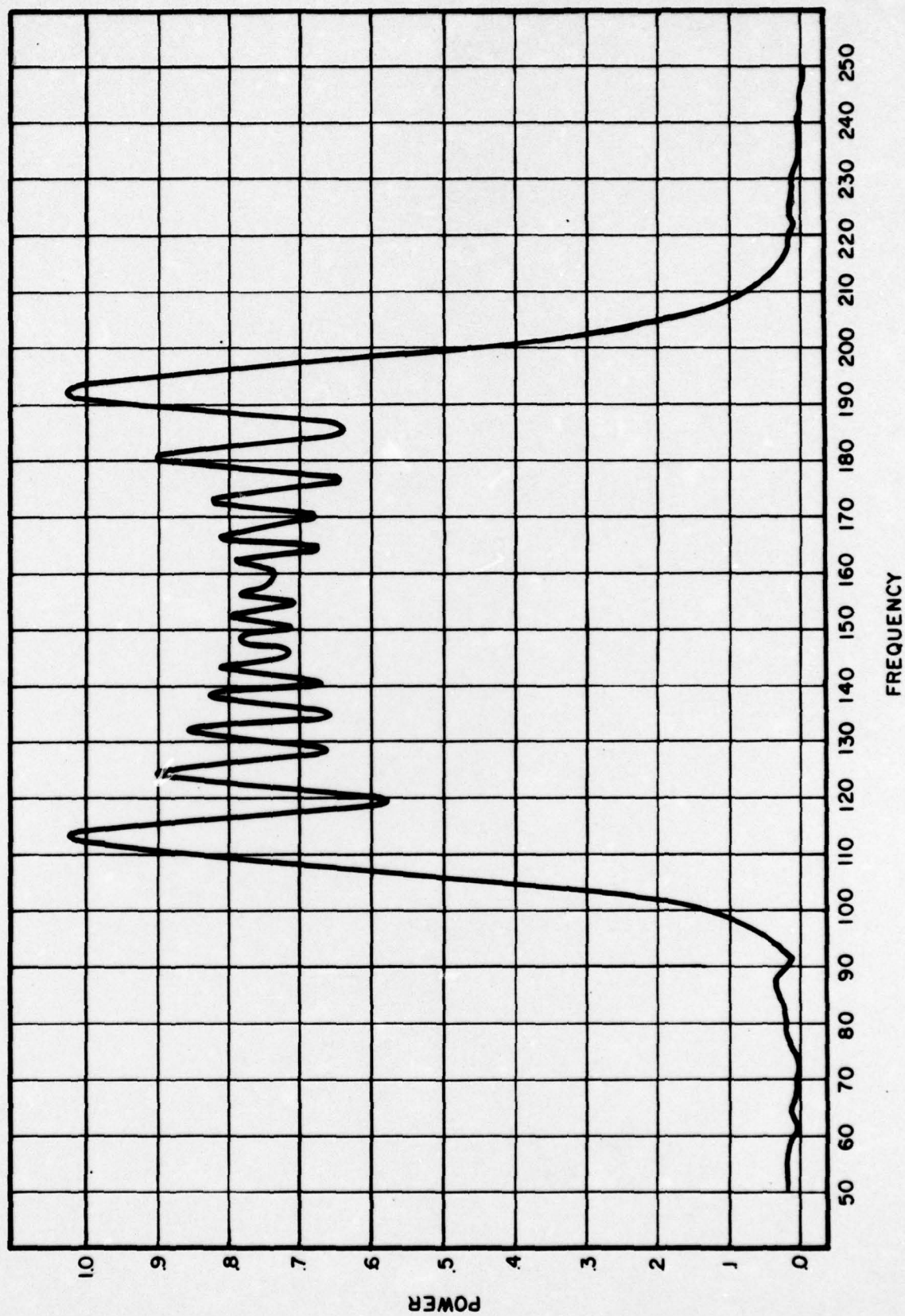


Fig. 6 - MEASURED POWER DENSITY SPECTRUM OF FM SLIDE GENERATED BY ALGORITHM

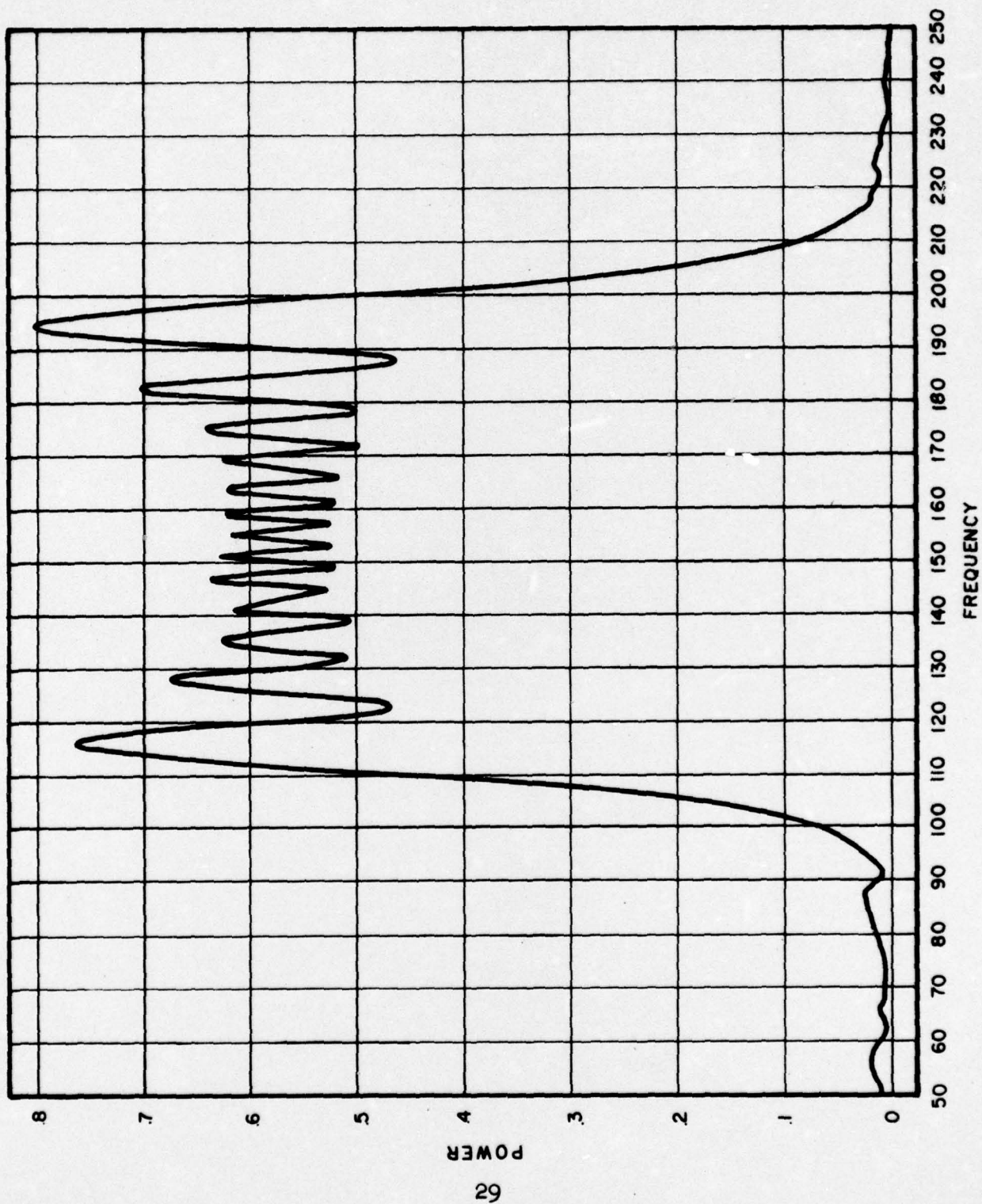


Fig. 7 - MEASURED POWER DENSITY SPECTRUM OF TRUE FM SLIDE

FREQUENCY SHIFT, cps

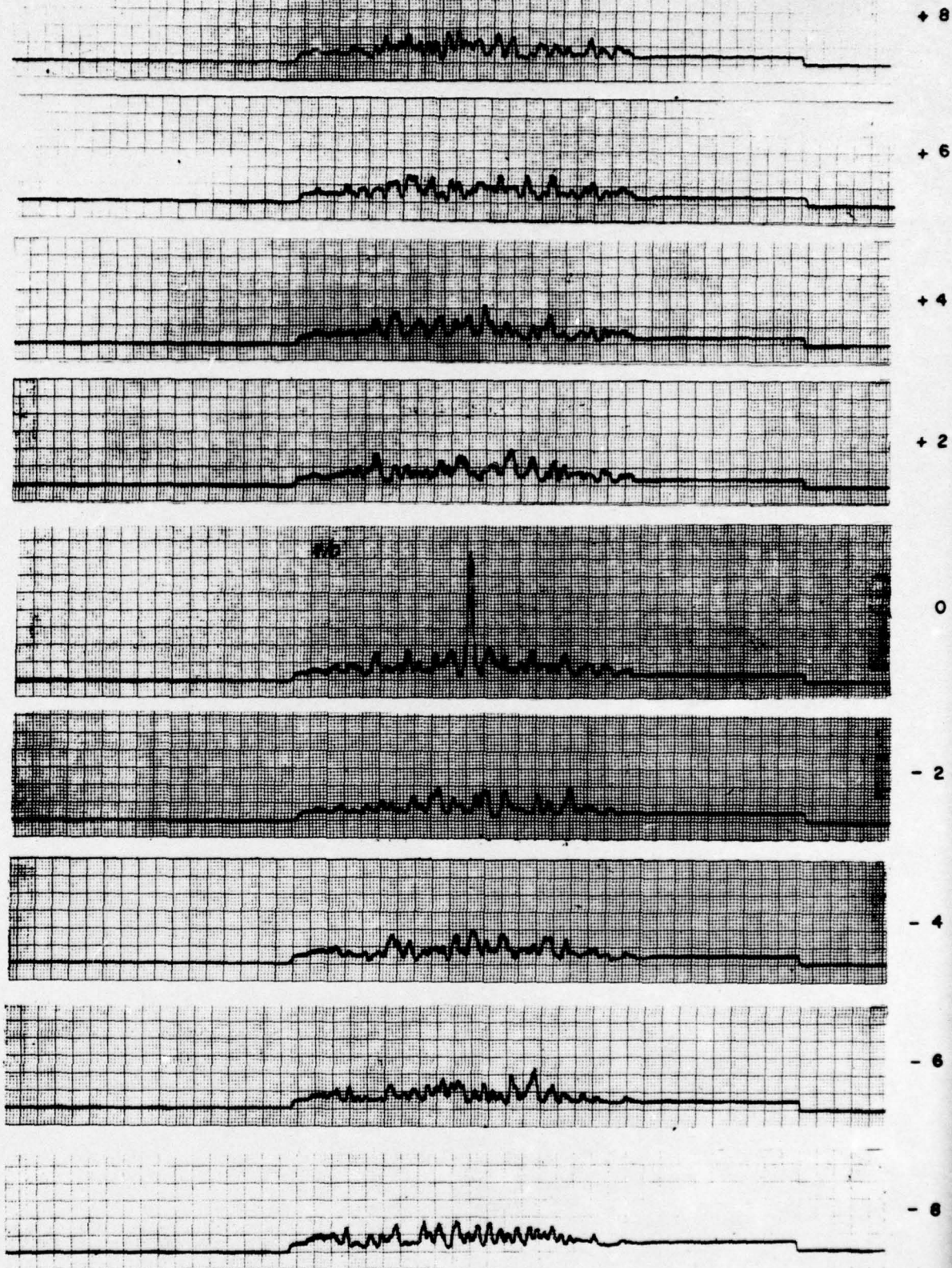


Fig.8-AMBIQUITY DIAGRAM OF PRN SIGNAL NUMBER 4

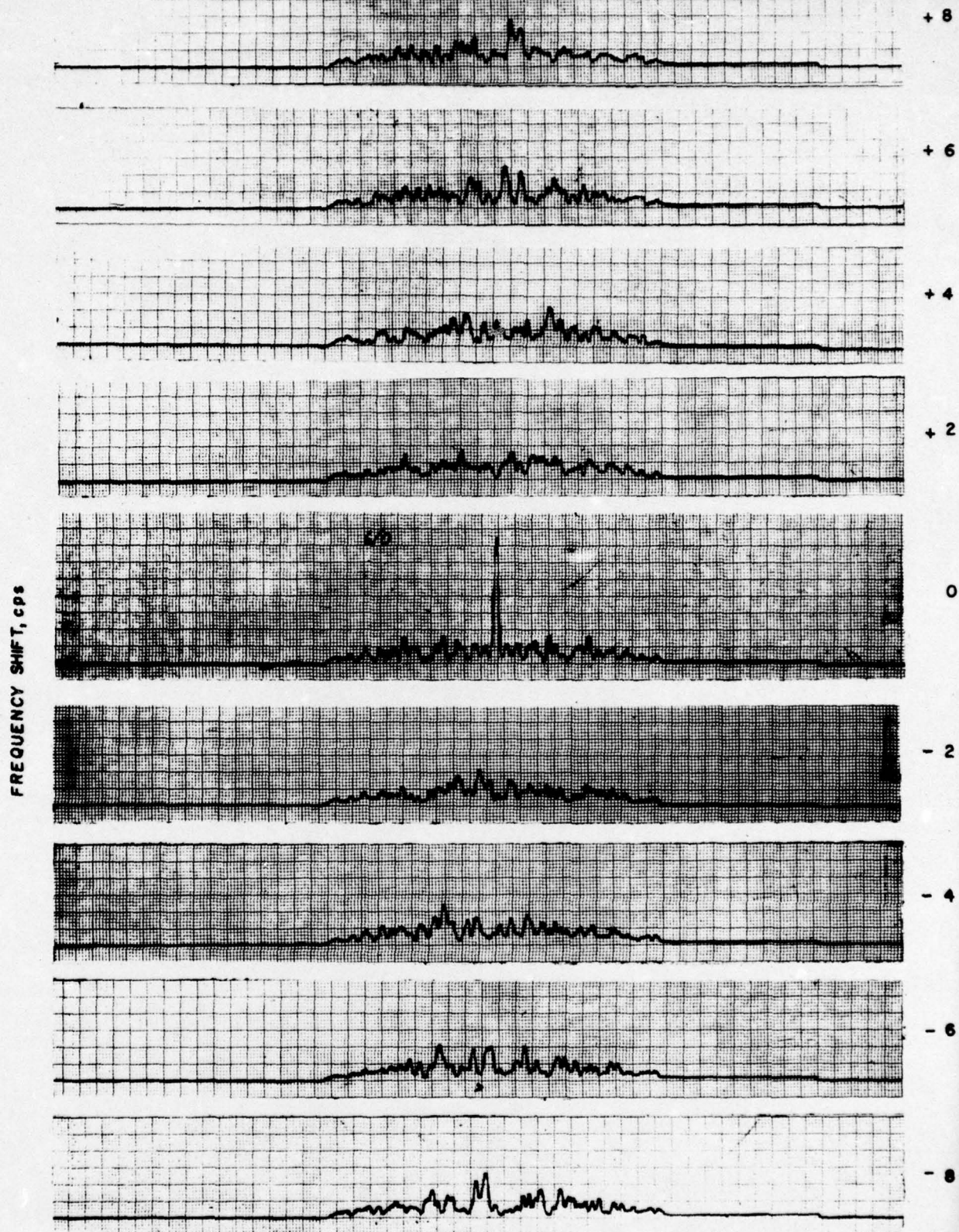


Fig. 9 - AMBIGUITY DIAGRAM OF PRN SIGNAL NUMBER 6

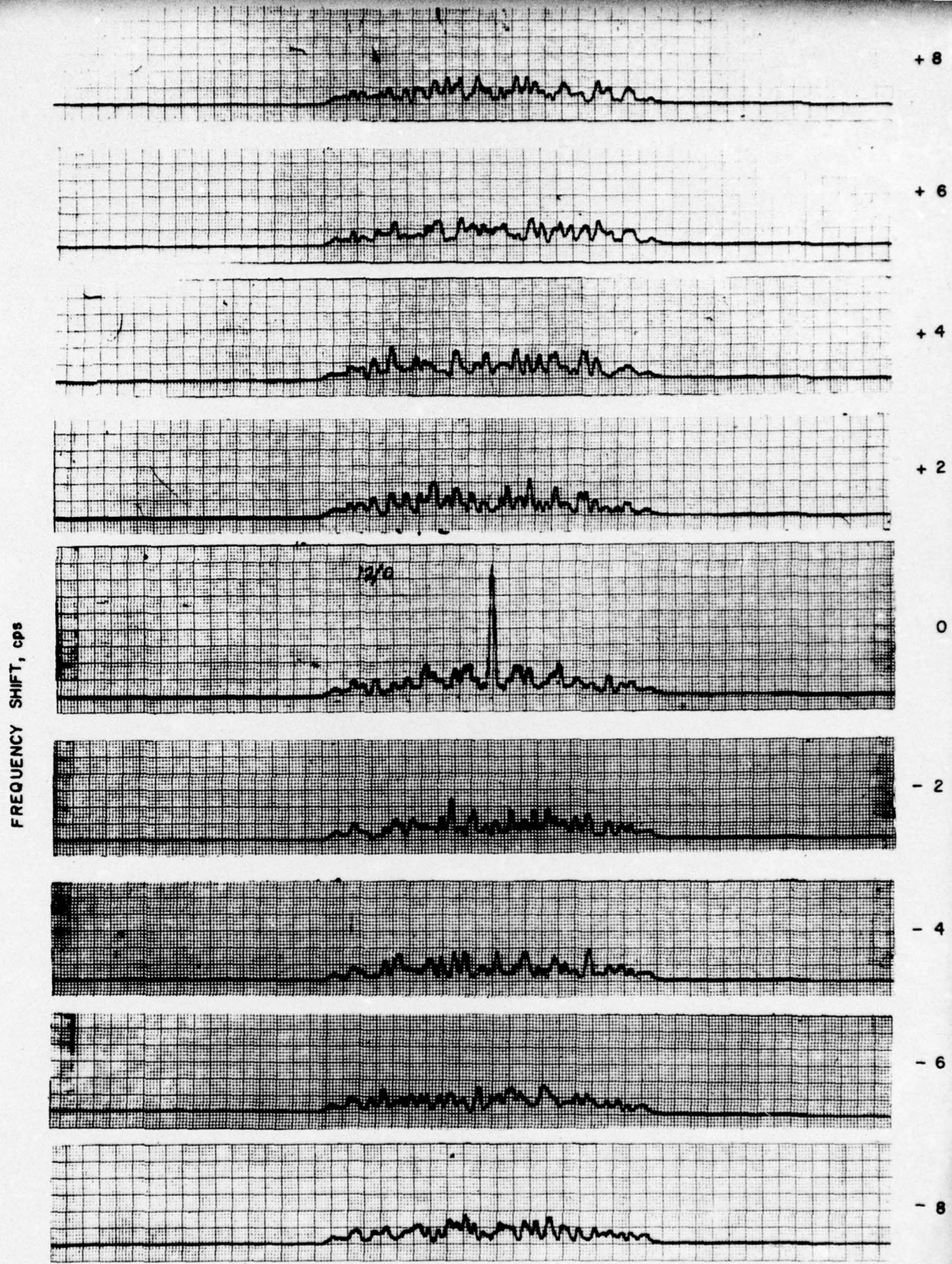


Fig.10-AMBIQUITY DIAGRAM OF PRN SIGNAL NUMBER 12

